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Mathematical Modelling of Benard-Marangoni Ferroconvection's Linear Stability in the Presence of Vertical Throughflow

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Abstract: Hypothetically, the stability study of throughflow influence on Bénard-Marangoni ferroconvection is examined. The top of the fluid layer is assumed to be free. The surface tension effect that depends on temperature is supposed to be non-deformable and subject to general thermal boundary conditions. The bottom of the fluid layer is assumed to be rigid with a fixed temperature. An analytical solution to the issue is achieved by using the Regular perturbation approach. The findings show that the stability characteristics are independent of the throughflow direction. The ferroconvection is further delayed by Peclet number Q. Convection is accelerated by raising the magnetic number Rm and the Prandtl number Pr. It is observed that the Bénard-Marangoni ferroconvection is unaffected by M3, which represents the non-linearity of fluid magnetization.

Keywords: Bénard-Marangoni Ferroconvection; Throughflow; Regular Perturbation Method.

1 Introduction

Synthetic magnetic fluids or ferrofluids are the colloidal suspensions of single domain nanoparticles (diameter is 3-10nm) of magnetite in non-conductive liquids such as water, heptane, kerosene etc. Because of their magnetic and liquid properties, these fluids came out as dependable materials for solving complex engineering problems. The authors [1], [2], [3] and many others provided an overview of this interesting topic with applications authoritatively. The study of convective instability in the ferrofluid layer was initiated by [4], and it was widely continued over the years (see references [5] to [11]).

Suppose the upper layer of the ferrofluid is open to the atmosphere. In that case, the instability is due to the combined effect of surface tension and buoyancy forces, known as Bénard-Marangoni ferroconvection. The convective instability due to these effects attracted the researchers. To begin with, the nonlinear and linear stability of buoyancy and surface tension effects in the ferrofluid layer is studied by [12]. The combination of Rosensweig and Marangoni instabilities by taking two semi-infinite immiscible and incompressible viscous fluids of infinite lateral level is studied by Weiplepp and Brand [13]. The authors [14] have explained the cause of initial temperature gradients on Marangoni ferroconvection to understand the control of ferroconvection. The instability of Bénard-Marangoni ferroconvection in the presence of an applied magnetic field due to various effects like MFD viscosity, internal heat generation and temperature-dependent viscosity was demonstrated by the authors [15], [16] and [17]. The Benard-Marangoni ferroconvection in a rotating ferrofluid layer with MFD viscosity was theoretically explained by [18].

The effect of throughflow on convection also draws considerable attention in the literature. The author was the first person who studies the instability in a porous intermediate by considering the case in which the convective effects rule the basic state temperature field on the throughflow. Later studied linear stability for small throughflow with both conducting boundaries, which are rigid and insulating. These authors showed that the convection is stabilized in the presence of throughflow. Authors [19] investigated the power of throughflow on convective instability in a porous medium by assuming that the boundaries are conducting and either permeable or impermeable. These works offered a physical explanation for the above situation and showed that destabilization occurs. The exact analysis of the throughflow effect on Marangoni convection was demonstrated. Authors [19] have theoretically explained the effect of throughflow on ferroconvection in a porous medium and explained double diffusive oscillatory convection with non-uniform heating effects in the porous medium. A similar study with variable viscosity and throughflow for ferromagnetic fluids was studied.

All the investigations above are limited, and no attempts have been made to understand the throughflow effect on control of Benard-Marangoni ferroconvection despite its significance in ferrofluid technology. Hence the present study examines the throughflow effect on Benard-Marangoni ferroconvection in the presence vertically applied magnetic field, and its nomenclatures are shown in Table 1.

Table 1: Nomenclature

$\vec{q}$	Velocity vector	ρ_0	Reference density at T_0
p	Pressure	k_t	Thermal conductivity
$\vec{H}$	Magnetic field intensity	χ	Magnetic susceptibility
$\vec{M}$	Magnetization	K	Pyromagnetic co-efficient
$\vec{B}$	Magnetic induction	ϕ	Magnetic potential
μ	Dynamic viscosity	∇^2	Laplacian
α_t	Co-efficient of thermal expansion	W	The amplitude of the vertical component of velocity
α	wave number	Θ	Amplitude of temperature
Ma	Marangoni number	Φ	The amplitude of the magnetic potential
T_b	Basic temperature	M_3	Non-linearity of magnetization parameter
T	Temperature	$\bar{T}$	Average temperature
∇_1^2	Horizontal Laplacian operator	Q	throughflow
M_1	Magnetic number	Pr	Prandtl number
R_t	Thermal Rayleigh number	R_m	Magnetic Rayleigh number

2 Formulation

We consider an inactive ferrofluid layer initially with an invariable throughflow of magnitude w_0 and gravity ($\vec{g} = -g\hat{k}$) associated in the direction with a vertically applied magnetic field H_0 as presented in the physical configuration. The bottom layer of fluid is taken as rigid, whereas the free upper surface is flat and non-deformable, where the surface tension effect is considered as $\sigma = \sigma_0 - \sigma_T(T - T_0)$ whereσ_0 is unperturbed value and $-\sigma_T$ rate of change of surface tension with temperature. The coordinates (x, y, z) are placed at the bottom layer with *z*–axis vertical as shown in figure 1.

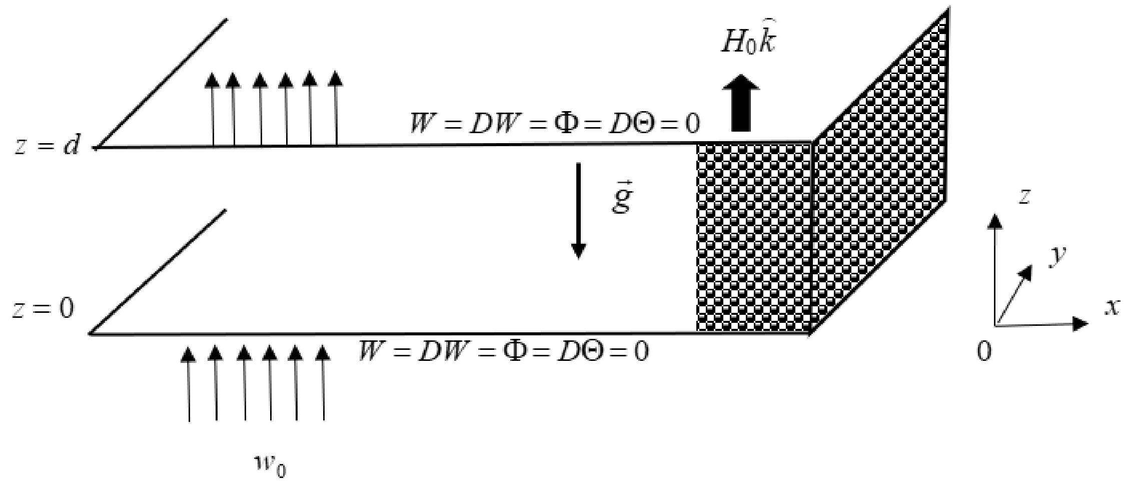

Figure 1: Physical Configuration

The principal equations of the Boussinesq approximation are

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right] = -\nabla p + \mu \nabla^2 \vec{q} + \rho_0 [1 - \alpha_t (T - T_0)]\vec{g} + \nabla \cdot (\vec{B}\vec{H}) \tag{2}$$

$$A \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = k_t \nabla^2 T \tag{3}$$

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{H} = 0 or \vec{H} = \nabla \phi \tag{4a, b}$$

$$\vec{B} = \mu_0 (\vec{M} + \vec{H}) \tag{5}$$

$$\vec{M} = [M_0 + \chi(H - H_0) - K(T - \bar{T})]\left(\frac{\vec{H}}{H}\right) \tag{6}$$

We assume that the fundamental state is inactive, and its solution is:

$$\overrightarrow{q_b}(z) = w_0 \hat{k}$$

$$p_b(z) = p_0 - \rho_0 g z - \frac{\rho_0 \alpha_t g \Delta T (w_0 z - \kappa e^{w_0 z/\kappa})}{w_0 (1 - e^{w_0 d/\kappa})} - \frac{\mu_0 K \Delta T e^{w_0 z/\kappa}(2 - e^{w_0 z/\kappa})}{2(1+\chi)(1 - e^{w_0 d/\kappa})^2}\left[M_0 + \frac{K\Delta T}{(1+\chi)}\right]$$

$$T_b(z) = T_0 - \Delta T \left[\frac{1 - e^{w_0 z/k}}{1 - e^{w_0 d/k}}\right]$$

$$\overrightarrow{H_b}(z) = \left[H_0 - \frac{K\Delta T}{1+\chi}\left(\frac{1 - e^{\frac{w_0 z}{k}}}{1 - e^{\frac{w_0 d}{k}}}\right)\right]\hat{k}$$

$$\overrightarrow{M_b}(z) = \left[M_0 - \frac{K\Delta T}{1+\chi}\left(\frac{1 - e^{\frac{w_0 z}{k}}}{1 - e^{\frac{w_0 d}{k}}}\right)\right]\hat{k} \tag{7}$$

Where, $\hat{k}$ is a unit vector along z - axis, and b denotes the fundamental state. Due to throughflow, temperature distribution at the fundamental state is nonlinear and has an intense effect on stability. But in the absence, the distribution at the basic state is linear and expressed as

$$T_b(z) = T_0 - \frac{\Delta T}{d} z$$

To examine the stability, we give small perturbations, as shown,

$$[\vec{q}, p, T, \vec{H}, \vec{M}] = [\omega_0 \hat{k} + \vec{q}', p_b(z) + p', T_b(z) + T', \vec{H}_b(z) + \vec{H}', \vec{M}_b(z) + \vec{M}'] \tag{8}$$

where, $\vec{q}', p', T', \vec{H}, \vec{M}'$ are small perturbation quantities.

Substituting (8) into (2), linearizing and operating curl twice to eliminate pressure term, a component of z in the subsequent equation is

$$\left[\rho_0 \frac{\partial}{\partial t} - \mu \nabla^2\right] \nabla^2 w = \rho_0 \alpha_t g \nabla_1^2 T + \frac{K\Delta T w_0 e^{w_0 z/\kappa}}{\kappa(1+\chi)(1 - e^{w_0 d/\kappa})}\left[\mu_0 (1+\chi)\frac{\partial}{\partial z}\nabla_1^2 \phi - \mu_0 K \nabla_1^2 T\right] \tag{9}$$

Substituting equation (8) in equation (3) and linearizing, we get

$$A\left(\frac{\partial T}{\partial t}+w_0\frac{\partial T}{\partial z}\right)=k_t\nabla^2T-\frac{{w_0}^2\Delta T}{\kappa(1-e^{w_0d/\kappa})}e^{w_0z/\kappa} \tag{10}$$

Equation (6), after substituting (8), may be written as

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\phi+(1+\chi)\frac{\partial^2\phi}{\partial z^2}-K\frac{\partial T}{\partial z}=0 \tag{11}$$

We assume that the stability exchange principle holds good, and hence we considered normal mode expansion as

$$\{w,T,\phi\}=\{W(z),\Theta(z),\Phi(z)\}e^{i(lx+my)} \tag{12}$$

Where, l,m represents wave numbers in x and y direction.

We substitute equation (12) into equations (9) to (11), non-dimensionalizing by choosing

$$(x*,y*,z*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),W*=\frac{d}{\nu}W,\Theta*=\frac{\kappa}{\beta\nu d}\Theta,\Phi*=\frac{(1+\chi)\,\kappa}{K\beta\nu d^2}\Phi \tag{13}$$

we obtain

$$(D^2-a^2)^2W-MD(D^2-a^2)W=-R_ma^2f(z)(D\Phi-\Theta)+R_ta^2\Theta \tag{14}$$

$$(D^2-a^2)\Theta-QD\Theta=f(z)W \tag{15}$$

$$(D^2-a^2M_3)\Phi-D\Theta=0 \tag{16}$$

where, $M=Q/Pr$ and $f(z)$ is given by,

$$f(z)=-\frac{Qe^{Qz}}{e^Q-1} \tag{17}$$

The following are the boundary conditions chosen to analyze the stability:

$$W=DW=\Phi=D\Theta=0 \qquad z=0$$

$$W=D^2W+Maa^2\Theta=D\Phi=D\Theta=0 \qquad z=1 \tag{18a,b}$$

3 Solution to the Problem

The Regular perturbation technique is applied to extract the decisive eigenvalues with perturbation parameter a (wave number). As a result, we expand W,Θ and Φ in powers of a^2 as

$$\{W(z),\Theta(z),\Phi(z)\}=\{W_0,\Theta_0,\Phi_0\}+a^2\{W_1,\Theta_1,\Phi_1\}+-------- \tag{19}$$

Substituting (19) into (14) to (16) and using the conditions (18a, b), gathering the coefficients of terms of order zero, we get

$$D^4W_0-MD^3W_0=0 \tag{20a}$$

$$D^2\Theta_0-QD\Theta_0-f(z)W_0=0 \tag{20b}$$

$$D^2\Phi_0-D\Theta_0=0 \tag{20c}$$

Solving the above equations, we obtain the solution to equations of order zero as

$$W_0=0,\Theta_0=1 and \Phi_0=0 \tag{21}$$

Similarly, equations of order one are,

$$D^4W_1-MD^3W_1=R_t[1+M_1f(z)] \tag{22a}$$

$$D^2\Theta_1-QD\Theta_1=1+f(z)W_1 \tag{22b}$$

$$D^2\Phi_1-D\Theta_1=0 \tag{22c}$$

Solving equation (22a), we get

$$W_1=c_1+c_2z+c_3z^2+c_4e^{Mz}-\frac{z^3R_t}{6M}+\frac{RM_1e^{Qz}}{(1-e^Q)(Q^3-MQ)} \tag{23}$$

where, $M=Q/Pr$ and $c_i^{'}s$ are constants of integration and are given by

$c_1 = \frac{1}{\gamma_2}\left[-\frac{R_t}{3M} + \frac{Ma}{2}\right] - \gamma_3 - \gamma_1,$

$c_2 = \frac{-M}{\gamma_2}\left[-\frac{R_t}{3M} + \frac{Ma}{2}\right] - M\gamma_3 - Q\gamma_1,$

$c_3 = \frac{R_t}{2M}\left[1 + \frac{M^2 e^M}{3\gamma_2}\right] - \frac{Ma}{2}\left[1 + \frac{M^2 e^M}{2\gamma_2}\right] - \frac{1}{2}\left[Q^2 e^Q \gamma_1 + M^2 e^M \gamma_3\right]$ and

$c_4 = \frac{1}{\gamma_2}\left[-\frac{R_t}{3M} + \frac{Ma}{2}\right] + \gamma_3$

where, $\gamma_1 = \frac{RM_1}{(1-e^Q)(Q^3 - MQ^2)}$

$\gamma_2 = 1 - M - \frac{M^2 e^M}{2} + e^M$ and

$\gamma_3 = \frac{\gamma_1}{\gamma_2}\left[1 + Q + \frac{Q^2 e^Q}{2} - e^Q\right]$

Integrating (22b) with respect to z, taking limits from 0 to 1, and using fundamental temperature conditions, we get

$$\int_0^1 f(z) W_1 dz = -1 \tag{24}$$

Substituting for W_1 from (23) and $f(z)$ from (17) into (24), carrying out the integration under the limiting conditions as → 0, leads to the expression.

$$R_{tc} = \frac{20(48 - Ma_c)}{3(1 + M_1)} \tag{25}$$

Further, as $Ma_c = 0$ Eq. (25) reduces to the form

$$R_{tc} = \frac{320}{1 + M_1} \tag{26}$$

This coincides with the expression of R_{tc} by [28]. For $M_1 = 0$ equation (26) gives $R_{tc} = 320$, which is the precise value for ordinary viscous fluids.

Further, if $R_{tc} = 0$ in equation (26), we obtained $Ma_c = 48$, the exact value of the critical Marangoni number for ordinary viscous fluids [20].

4 Experimental Results

The approach used in this paper successfully performed a two-stage classification of bugs reported from the execution of Chef cookbooks. Comparing various machine learning techniques and deep learning algorithms found that the deep learning CNN algorithm gives the highest accuracy score. Since the second classification layer depends on the first layer, having a high-accuracy model in the first layer is the most important aspect. Fig. 2 compares various classifiers in the first stage of classification. The graph in Fig. 2 shows that CNN achieves the highest accuracy score of 97.73 percent compared to the other algorithms, whose accuracy score is less than 90 percent for the same data. The naïve Bayes algorithm yielded an accuracy of 85 percent. The decision tree algorithm classified 76 percent accurately compared to the 86 percent of Random Forest. The performance of SVM surpassed that of the other machine learning algorithms with an accuracy of 87 percent.

The critical eigenvalues R_{tc} and Ma_c for various values of $Rm, Pr,$ and $|Q|$ are computed analytically. It is found that the stability characters of the system are free of M_3. Significant features of these parameters are shown graphically in Figs. 2-8.

In the existence of throughflow, magnetic field, magnetization and temperature at basic state will diverge from linear to nonlinear along a vertical direction, which has considerable influence on the stability (see Fig. 2). To review the impact of throughflow on ferroconvection, the dimensionless fundamental state distributions $\tilde{T}_b(z)$,$\widetilde{H}_b(z)$ and $\widetilde{M}_b(z)$ for diverse values of $|Q|$ are plotted graphically in Fig 2. From the figure, it is evident that increasing throughflow direction results in hefty deviations in these scatterings which in turn augment the instability in the ferrofluid layer.

In Figures 3 and 4, we have studied the convective instability only due to buoyancy forces. Fig. 3 and 4 represent the deviation of R_{tc} versus $|Q|$ for diverse values of Rm (see Fig.3) and Prandtl number Pr (see Fig. 4) in the absence of surface tension effects. From fig. 3 it is clear that as Rm increases R_{tc} decreases as expected and makes the system more unstable. This is because additive support of destabilizing magnetic force enhances the onset of ferroconvection. An analogous state prevails in the absence of thermal buoyancy forces (i.e.$R_t = 0$), and this case corresponds to Marangoni ferroconvection (see Fig. 5). Further, the direction of throughflow does not change the system stability system and as $|Q|$ increases both R_{tc} and Ma_c also increase. Fig.4 presents the deviation of R_{tc} versus $|Q|$ for various values of Pr. The results show that in the absence of surface tension force, increasing in Prandtl number Pr does not significantly affect the onset of ferroconvection. Fig. 6, shows that as Pr increases, and Ma decreases. Hence, its effect is to hasten the ferroconvection.

Figure 7 shows convective instability with both buoyancy and surface tension forces. A plot R_{tc} versus Ma_c is shown in Fig.7 for different M_1 with $|Q| = 5$ and $Pr = 10$. From the graphical representation, it is clear that there is a strong coupling between R_{tc} and Ma_c. When the surface tension forces are strong, the buoyancy force becomes negligible and vice-versa. Also, we observe that as M_1 increases, the destabilizing magnetic force also increases and hence hastens the ferroconvection. Further, the curves for different M_1 converge to $Ma_c = 48$ when $R_{tc} = 0$ demonstrating that it doesn't affect Marangoni ferroconvection. Theoretically, the results support this behaviour (refer (26)). Fig. 8 shows that as $|Q|$ increases both Ma_c and R_{tc} also increases. From the figure, we observe that increasing $|Q|$ Stabilizes the system. Thus, it is observed that adjusting vertical throughflow can control the onset of ferroconvection.

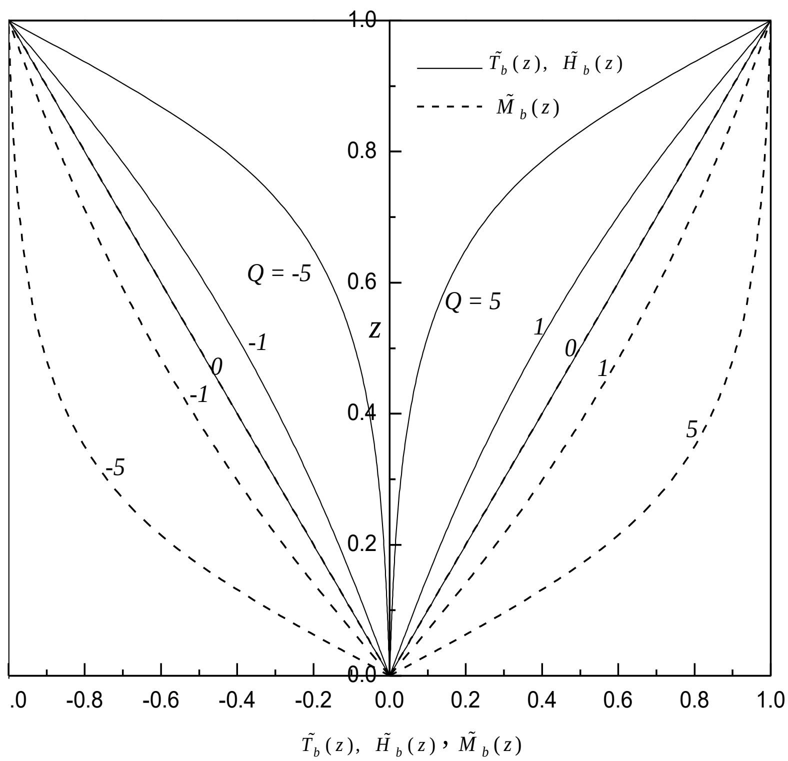

Figure 2: Distribution of basic profiles for different values of Q

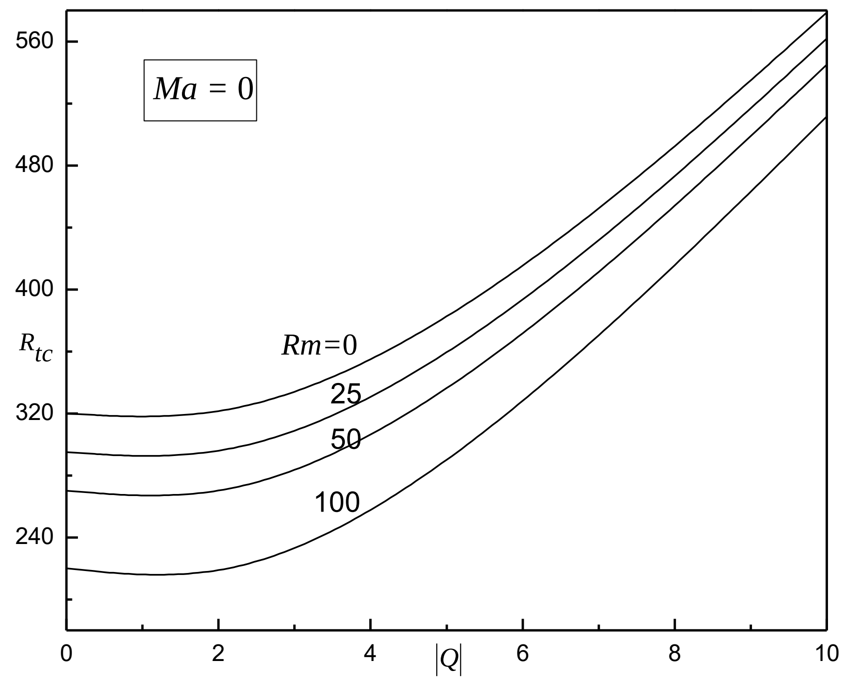

Figure 3: Variation of R_{tc} against $|Q|$ for different values of Rm

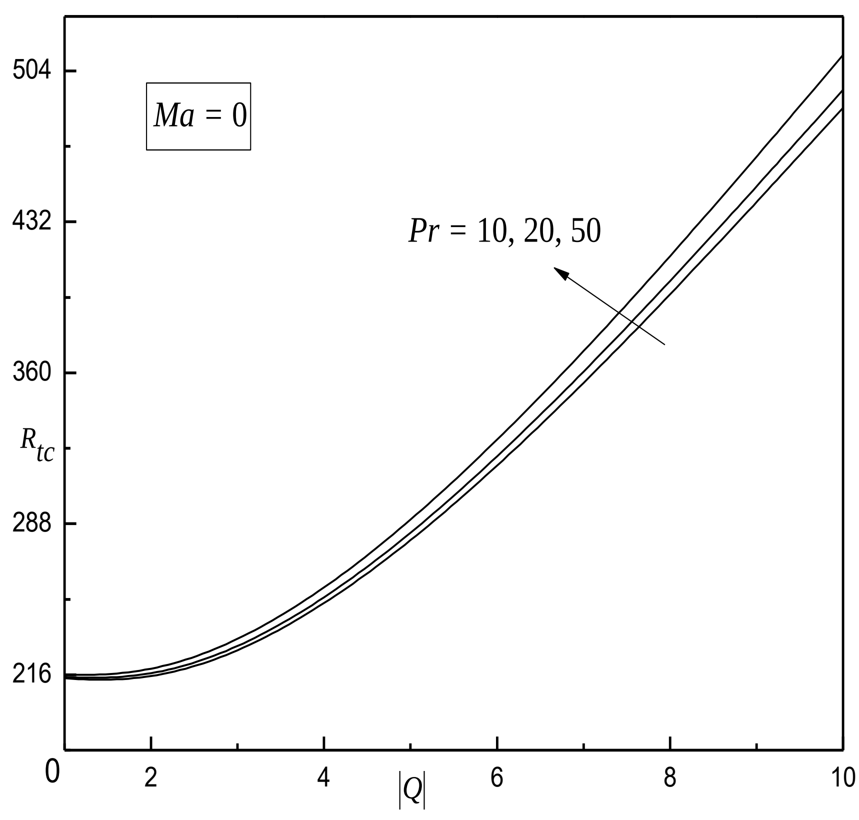

Figure 4: Variation of R_{tc} against $|Q|$ for different values of Pr

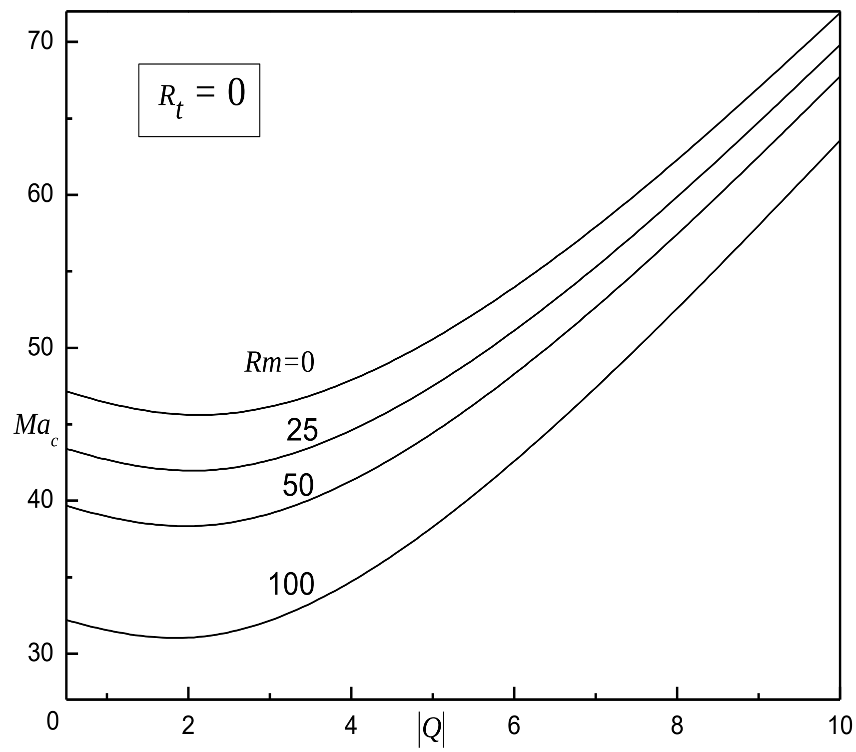

Figure 5: Variation of Ma_c against $|Q|$ for different values of Rm

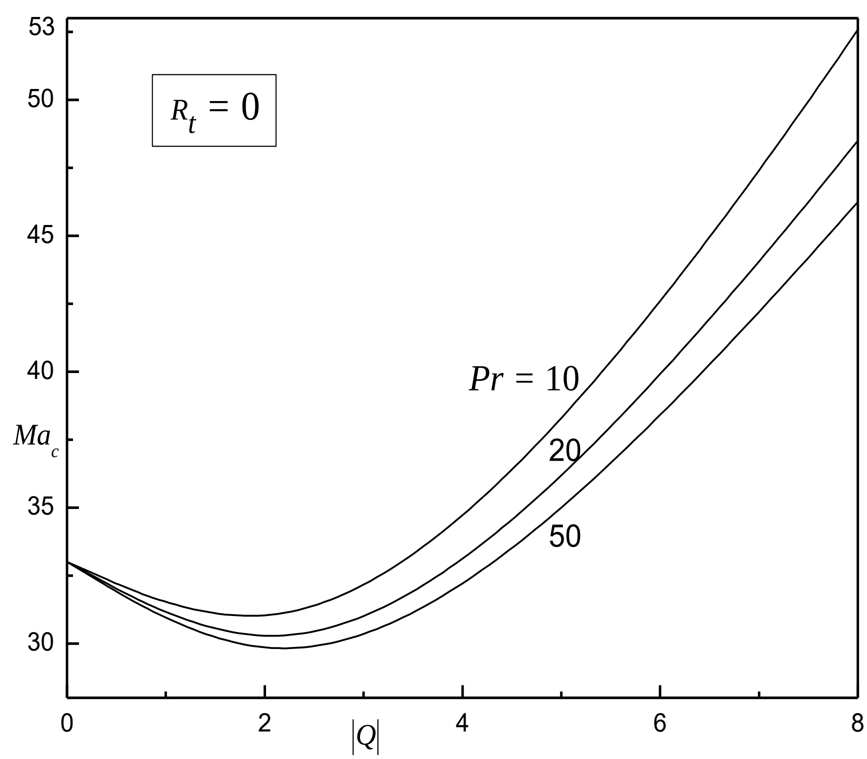

Figure 6: Variation of Ma_c against $|Q|$ for different values of Pr

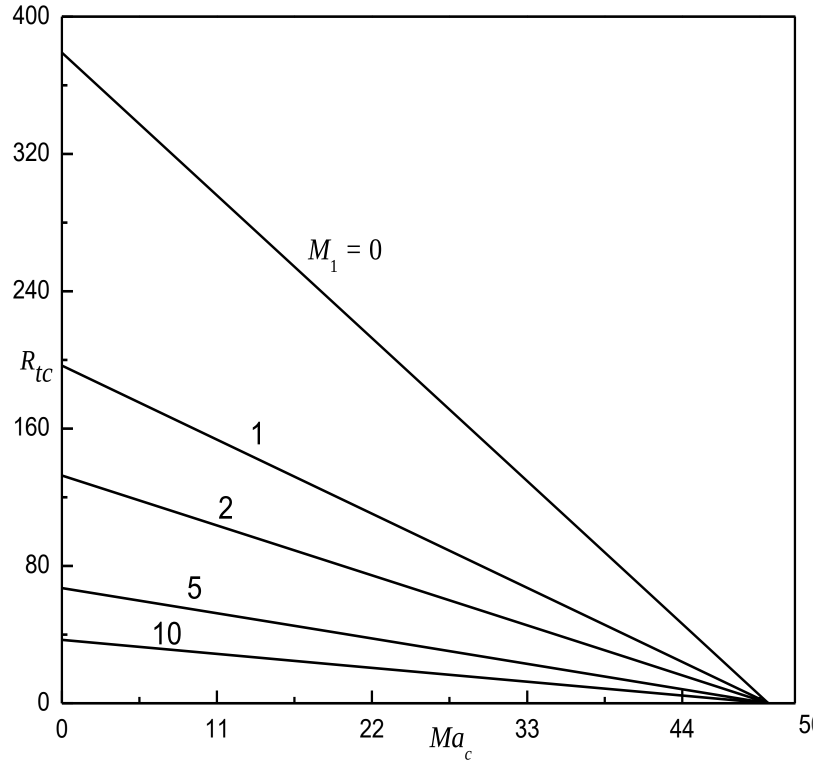

Figure 7: Variation of R_{tc} against Ma_c for different values of M_1 when $|Q| = 5, Pr = 10$.

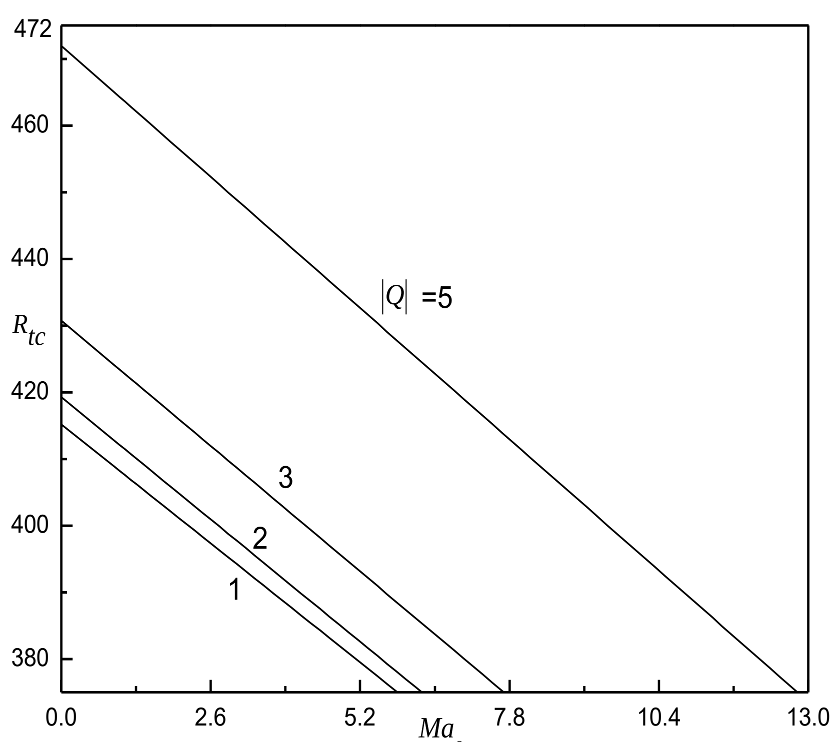

Figure 8: Variation of R_{tc} against Ma_c for different values of $|Q|$ when $Pr = 10, Rm = 100$.

5 Conclusion and Future Scope

From the above study, we can conclude,

(i) The basic state distributions $T_b(z), \overrightarrow{H_b}(z)$ and $\overrightarrow{M_b}(z)$ are nonlinear in the presence of vertical throughflow $|Q|$ and it effect the stability significantly.

(ii) The increase in the values of Rm and Pr is to speed up the ferroconvection. While, M_3 does not have any effect on the ferroconvection.

(iii) The effect of Peclet number Q, depending on throughflow, delays the inception of ferroconvection.

(iv) Magnetic and buoyancy forces strengthen one another and augment the ferroconvection.

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